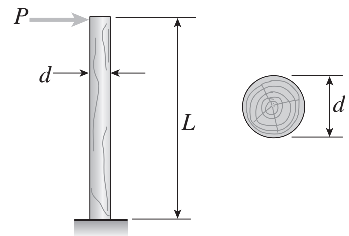
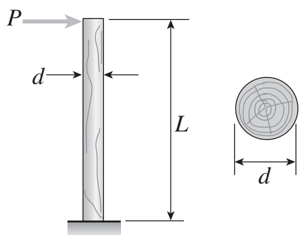
## Shear Stresses in Circular Beams

**Problem 5.9-1** A wood pole of solid circular cross section ($d$ = diameter) is subjected to a horizontal force $P = 450$ lb (see figure). The length of the pole is $L = 6$ ft, and the allowable stresses in the wood are 1900 psi in bending and 120 psi in shear.

Determine the minimum required diameter of the pole based upon (a) the allowable bending stress, and (b) the allowable shear stress.




**Solution 5.9-1** **Wood pole of circular cross section**




$P = 450$ lb $\quad L = 6$ ft $= 72$ in.
$\sigma_{\text{allow}} = 1900$ psi
$\tau_{\text{allow}} = 120$ psi
Find diameter $d$

(a) Based upon bending stress

$M_{\max} = PL = (450\text{ lb})(72\text{ in.}) = 32{,}400$ lb-in.

$$\sigma = \frac{M}{S} = \frac{32M}{\pi d^3} \qquad d^3 = \frac{32M_{\max}}{\pi \sigma_{\text{allow}}} = 173.7\text{ in.}^3$$

$d_{\min} = 5.58$ in. ⟵

(b) Based upon shear stress

$V_{\max} = 450$ lb

$$\tau = \frac{4V}{3A} = \frac{16V}{3\pi d^2} \qquad d^2 = \frac{16V_{\max}}{3\pi \tau_{\text{allow}}} = 6.366\text{ in.}^2$$

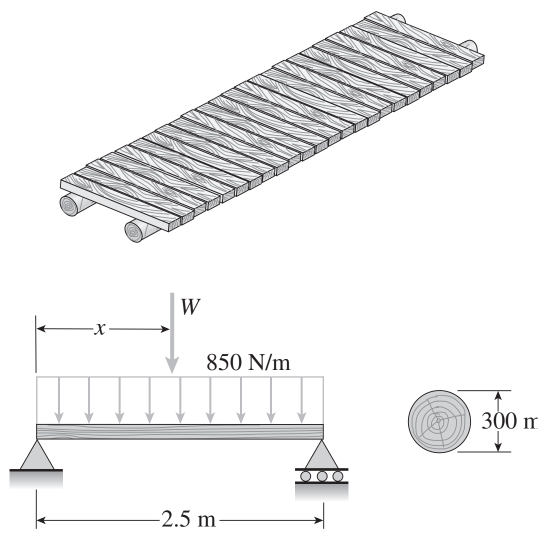
$d_{\min} = 2.52$ in. ⟵

(Bending stress governs.)

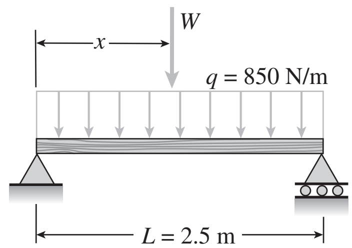
**Problem 5.9-2** A simple log bridge in a remote area consists of two parallel logs with planks across them (see figure). The logs are Douglas fir with average diameter 300 mm. A truck moves slowly across the bridge, which spans 2.5 m. Assume that the weight of the truck is equally distributed between the two logs.

Because the wheelbase of the truck is greater than 2.5 m, only one set of wheels is on the bridge at a time. Thus, the wheel load on one log is equivalent to a concentrated load $W$ acting at any position along the span. In addition, the weight of one log and the planks it supports is equivalent to a uniform load of 850 N/m acting on the log.

Determine the maximum permissible wheel load $W$ based upon (a) an allowable bending stress of 7.0 MPa, and (b) an allowable shear stress of 0.75 MPa.



**Solution 5.9-2** **Log bridge**




Diameter $d = 300$ mm
$\sigma_{allow} = 7.0$ MPa
$\tau_{allow} = 0.75$ MPa
Find allowable load $W$

(a) Based upon bending stress
Maximum moment occurs when wheel is at midspan ($x = L/2$).

$$M_{max} = \frac{WL}{4} + \frac{qL^2}{8} = \frac{W}{4}(2.5 \text{ m}) + \frac{1}{8}(850 \text{ N/m})(2.5 \text{ m})^2$$
$$= 0.625W + 664.1 \text{ (N} \cdot \text{m)} \quad (W = \text{newtons})$$

$$S = \frac{\pi d^3}{32} = 2.651 \times 10^{-3} \text{ m}^3$$
$$M_{max} = S\sigma_{allow} = (2.651 \times 10^{-3} \text{ m}^3)(7.0 \text{ MPa}) = 18{,}560 \text{ N} \cdot \text{m}$$

$\therefore 0.625W + 664.1 = 18{,}560$
$W = 28{,}600 \text{ N} = 28.6 \text{ kN}$ ←

(b) Based upon shear stress
Maximum shear force occurs when wheel is adjacent to support ($x = 0$).

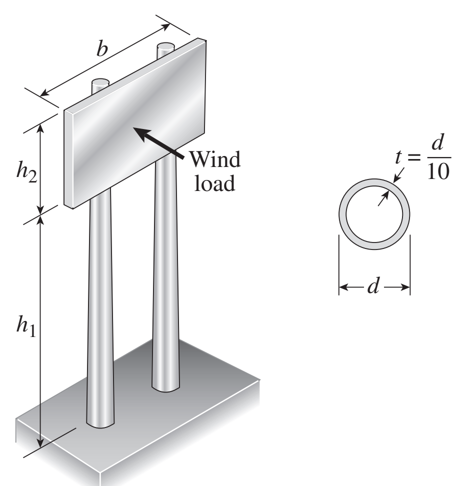
$$V_{max} = W + \frac{qL}{2} = W + \frac{1}{2}(850 \text{ N/m})(2.5 \text{ m})$$
$$= W + 1062.5 \text{ N} \quad (W = \text{newtons})$$
$$A = \frac{\pi d^2}{4} = 0.070686 \text{ m}^2$$
$$\tau_{max} = \frac{4V_{max}}{3A}$$
$$V_{max} = \frac{3A\,\tau_{allow}}{4} = \frac{3}{4}(0.070686 \text{ m}^2)(0.75 \text{ MPa})$$
$$= 39{,}760 \text{ N}$$

$\therefore W + 1062.5 \text{ N} = 39{,}760 \text{ N}$
$W = 38{,}700 \text{ N} = 38.7 \text{ kN}$ ←

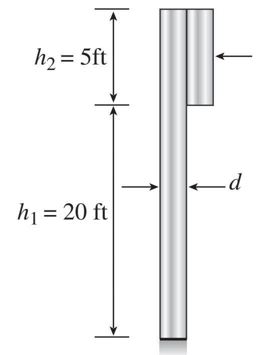
**Problem 5.9-3** A sign for an automobile service station is supported by two aluminum poles of hollow circular cross section, as shown in the figure. The poles are being designed to resist a wind pressure of 75 lb/ft$^2$ against the full area of the sign. The dimensions of the poles and sign are $h_1 = 20$ ft, $h_2 = 5$ ft, and $b = 10$ ft. To prevent buckling of the walls of the poles, the thickness $t$ is specified as one-tenth the outside diameter $d$.

(a) Determine the minimum required diameter of the poles based upon an allowable bending stress of 7500 psi in the aluminum.

(b) Determine the minimum required diameter based upon an allowable shear stress of 2000 psi.



## Solution 5.9-3 Wind load on a sign




**Probs. 5.9-3 and 5.9-4**

$b$ = width of sign
$b = 10$ ft
$P = 75$ lb/ft$^2$
$\sigma_{allow} = 7500$ psi
$\tau_{allow} = 2000$ psi
$d$ = diameter   $W$ = wind force on one pole

$$t = \frac{d}{10} \qquad W = ph_2\left(\frac{b}{2}\right) = 1875 \text{ lb}$$

(a) Required diameter based upon bending stress

$$M_{max} = W\left(h_1 + \frac{h_2}{2}\right) = 506{,}250 \text{ lb-in.}$$

$$I = \frac{\pi}{64}(d_2^4 - d_2^4) \quad d_2 = d \quad d_1 = d - 2t = \frac{4}{5}d$$

$$I = \frac{\pi}{64}\left[d^4 - \left(\frac{4d}{5}\right)^4\right] = \frac{\pi d^4}{64}\left(\frac{369}{625}\right) = \frac{369\pi d^4}{40{,}000} \text{ (in.}^4\text{)}$$

$$c = \frac{d}{2} \quad (d = \text{inches})$$

$$\sigma = \frac{Mc}{I} = \frac{M(d/2)}{369\pi d^4/40{,}000} = \frac{17.253M}{d^3}$$

$$d^3 = \frac{17.253M_{max}}{\sigma_{allow}} = \frac{(17.253)(506{,}250 \text{ lb-in.})}{7500 \text{ psi}}$$

$$= 1164.6 \text{ in.}^3 \quad d = 10.52 \text{ in.} \leftarrow$$

(b) Required diameter based upon shear stress

$$V_{max} = W = 1875 \text{ lb}$$

$$\tau = \frac{4V}{3A}\left(\frac{r_2^2 + r_2r_1 + r_1^2}{r_2^2 + r_1^2}\right) \quad r_2 = \frac{d}{2}$$

$$r_1 = \frac{d}{2} - t = \frac{d}{2} - \frac{d}{10} = \frac{2d}{5}$$

$$\frac{r_2^2 + r_2r_1 + r_1^2}{r_2^2 + r_1^2} = \frac{\left(\frac{d}{2}\right)^2 + \left(\frac{d}{2}\right)\left(\frac{2d}{5}\right) + \left(\frac{2d}{5}\right)^2}{\left(\frac{d}{2}\right)^2 + \left(\frac{2d}{5}\right)^2} = \frac{61}{41}$$

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}\left[d^2 - \left(\frac{4d}{5}\right)^2\right] = \frac{9\pi d^2}{100}$$

$$\tau = \frac{4V}{3}\left(\frac{61}{41}\right)\left(\frac{100}{9\pi d^2}\right) = 7.0160\frac{V}{d^2}$$

$$d^2 = \frac{7.0160\,V_{max}}{\tau_{allow}} = \frac{(7.0160)(1875 \text{ lb})}{2000 \text{ psi}} = 6.5775 \text{ in.}^2$$

$d = 2.56$ in. ←
(Bending stress governs.)

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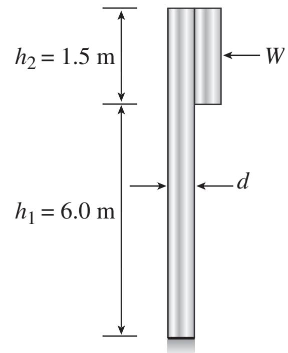
**Problem 5.9-4** Solve the preceding problem for a sign and poles having the following dimensions: $h_1 = 6.0$ m, $h_2 = 1.5$ m, $b = 3.0$ m, and $t = d/10$. The design wind pressure is 3.6 kPa, and the allowable stresses in the aluminum are 50 MPa in bending and 14 MPa in shear.

## Solution 5.9-4 Wind load on a sign




$b$ = width of sign

$b = 3.0$ m

$P = 3.6$ kPa

$\sigma_{allow} = 50$ MPa

$\tau_{allow} = 16$ MPa

$d$ = diameter $\quad W$ = wind force on one pole

$$t = \frac{d}{10} \qquad W = ph_2\left(\frac{b}{2}\right) = 8.1 \text{ kN}$$

(a) Required diameter based upon bending stress

$$M_{max} = W\left(h_1 + \frac{h_2}{2}\right) = 54.675 \text{ kN} \cdot \text{m}$$

$$\sigma = \frac{Mc}{I} \qquad I = \frac{\pi}{64}(d_2^4 - d_1^4) \qquad d_2 = d \qquad d_1 = d - 2t = \frac{4}{5}d$$

$$I = \frac{\pi}{64}\left[d^4 - \left(\frac{4d}{5}\right)^4\right] = \frac{\pi d^4}{64}\left(\frac{369}{625}\right) = \frac{369\pi d^4}{40{,}000} \text{ (m}^4\text{)}$$

$$c = \frac{d}{2} \qquad (d = \text{meters})$$

$$\sigma = \frac{Mc}{I} = \frac{M(d/2)}{369\pi d^4/40{,}000} = \frac{17.253M}{d^3}$$

$$d^3 = \frac{17.253M_{max}}{\sigma_{allow}} = \frac{(17.253)(54.675 \text{ kN} \cdot \text{m})}{50 \text{ MPa}}$$

$$= 0.018866 \text{ m}^3$$

$d = 0.266$ m $= 266$ mm ←

(b) Required diameter based upon shear stress

$$V_{max} = W = 8.1 \text{ kN}$$

$$\tau = \frac{4V}{3A}\left(\frac{r_2^2 + r_1r_2 + r_1^2}{r_2^2 + r_1^2}\right) \qquad r_2 = \frac{d}{2}$$

$$r_1 = \frac{d}{2} - t = \frac{d}{2} - \frac{d}{10} = \frac{2d}{5}$$

$$\frac{r_2^2 + r_1r_2 + r_1^2}{r_2^2 + r_1^2} = \frac{\left(\frac{d}{2}\right)^2 + \left(\frac{d}{2}\right)\left(\frac{2d}{5}\right) + \left(\frac{2d}{5}\right)^2}{\left(\frac{d}{2}\right)^2 + \left(\frac{2d}{5}\right)^2} = \frac{61}{41}$$

$$A = \frac{\pi}{4}(d_2^2 + d_1^2) = \frac{\pi}{4}\left[d^2 - \left(\frac{4d}{5}\right)^2\right] = \frac{9\pi d^2}{100}$$

$$\tau = \frac{4V}{3}\left(\frac{61}{41}\right)\left(\frac{100}{9\pi d^2}\right) = 7.0160\frac{V}{d^2}$$

$$d^2 = \frac{7.0160\, V_{max}}{\tau_{allow}} = \frac{(7.0160)(8.1 \text{ kN})}{14 \text{ MPa}}$$

$$= 0.004059 \text{ m}^2$$

$d = 0.06371$ m $= 63.7$ mm ←

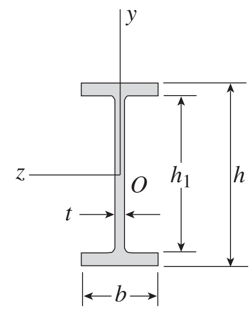
(Bending stress governs)

## Shear Stresses in Circular Beams with Flanges

**Problem 5.10-1 through 5.10-6** A wide-flange beam (see figure) having the cross section described below is subjected to a shear force $V$. Using the dimensions of the cross section, calculate the moment of inertia and then determine the following quantities:

(a) The maximum shear stress $\tau_{max}$ in the web.
(b) The minimum shear stress $\tau_{min}$ in the web.
(c) The average shear stress $\tau_{aver}$ (obtained by dividing the shear force by the area of the web) and the ratio $\tau_{max}/\tau_{aver}$.
(d) The shear force $V_{web}$ carried in the web and the ratio $V_{web}/V$.

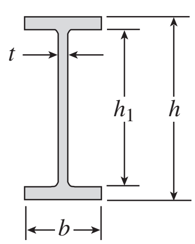
*Note:* Disregard the fillets at the junctions of the web and flanges and determine all quantities, including the moment of inertia, by considering the cross section to consist of three rectangles.



**Probs. 5.10-1 through 5.10-6**

**Problem 5.10-1** Dimensions of cross section: $b = 6$ in., $t = 0.5$ in., $h = 12$ in., $h_1 = 10.5$ in., and $V = 30$ k.

**Solution 5.10-1 Wide-flange beam**




$b = 6.0$ in.
$t = 0.5$ in.
$h = 12.0$ in.
$h_1 = 10.5$ in.
$V = 30$ k

Moment of inertia (Eq. 5-47)

$$I = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3) = 333.4 \text{ in.}^4$$

(a) Maximum shear stress in the web (Eq. 5-48a)

$$\tau_{max} = \frac{V}{8It}(bh^2 - bh_1^2 + th_1^2) = 5795 \text{ psi} \longleftarrow$$

(b) Minimum shear stress in the web (Eq. 5-48b)

$$\tau_{min} = \frac{Vb}{8It}(h^2 - h_1^2) = 4555 \text{ psi} \longleftarrow$$

(c) Average shear stress in the web (Eq. 5-50)

$$\tau_{aver} = \frac{V}{th_1} = 5714 \text{ psi} \longleftarrow$$

$$\frac{\tau_{max}}{\tau_{aver}} = 1.014 \longleftarrow$$

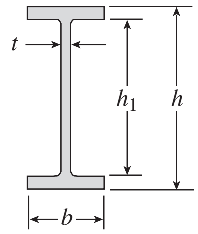
(d) Shear force in the web (Eq. 5-49)

$$V_{web} = \frac{th_1}{3}(2\tau_{max} + \tau_{min}) = 28.25 \text{ k} \longleftarrow$$

$$\frac{V_{web}}{V} = 0.942 \longleftarrow$$

**Problem 5.10-2** Dimensions of cross section: $b = 180$ mm, $t = 12$ mm, $h = 420$ mm, $h_1 = 380$ mm, and $V = 125$ kN.

**Solution 5.10-2** **Wide-flange beam**




$b = 180$ mm
$t = 12$ mm
$h = 420$ mm
$h_1 = 380$ mm
$V = 125$ kN

MOMENT OF INERTIA (Eq. 5-47)

$$I = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3) = 343.1 \times 10^6 \text{ mm}^4$$

(a) MAXIMUM SHEAR STRESS IN THE WEB (Eq. 5-48a)

$$\tau_{max} = \frac{V}{8It}(bh^2 - bh_1^2 + th_1^2) = 28.43 \text{ MPa} \longleftarrow$$

(b) MINIMUM SHEAR STRESS IN THE WEB (Eq. 5-48b)

$$\tau_{min} = \frac{Vb}{8It}(h^2 - h_1^2) = 21.86 \text{ MPa} \longleftarrow$$

(c) AVERAGE SHEAR STRESS IN THE WEB (Eq. 5-50)

$$\tau_{aver} = \frac{V}{th_1} = 27.41 \text{ MPa} \longleftarrow$$

$$\frac{\tau_{max}}{\tau_{aver}} = 1.037 \longleftarrow$$

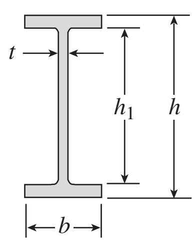
(d) SHEAR FORCE IN THE WEB (Eq. 5-49)

$$V_{web} = \frac{th_1}{3}(2\tau_{max} + \tau_{min}) = 119.7 \text{ kN} \longleftarrow$$

$$\frac{V_{web}}{V} = 0.957 \longleftarrow$$

**Problem 5.10-3** Wide-flange shape, W 8 × 28 (see Table E-1, Appendix E); $V = 10$ k.

**Solution 5.10-3** **Wide-flange beam**




W 8 × 28
$b = 6.535$ in.
$t = 0.285$ in.
$h = 8.06$ in.
$h_1 = 7.13$ in.
$V = 10$ k

MOMENT OF INERTIA (Eq. 5-47)

$$I = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3) = 96.36 \text{ in.}^4$$

(a) MAXIMUM SHEAR STRESS IN THE WEB (Eq. 5-48a)

$$\tau_{max} = \frac{V}{8It}(bh^2 - bh_1^2 + th_1^2) = 4861 \text{ psi} \longleftarrow$$

(b) MINIMUM SHEAR STRESS IN THE WEB (Eq. 5-48b)

$$\tau_{min} = \frac{Vb}{8It}(h^2 - h_1^2) = 4202 \text{ psi} \longleftarrow$$

(c) AVERAGE SHEAR STRESS IN THE WEB (Eq. 5-50)

$$\tau_{aver} = \frac{V}{th_1} = 4921 \text{ psi} \longleftarrow$$

$$\frac{\tau_{max}}{\tau_{aver}} = 0.988 \longleftarrow$$

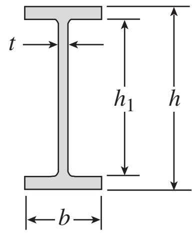
(d) Shear force in the web (Eq. 5-49)

$$V_{web} = \frac{th_1}{3}(2\tau_{max} + \tau_{min}) = 9.432 \text{ k} \longleftarrow$$

$$\frac{V_{web}}{V} = 0.943 \longleftarrow$$

**Problem 5.10-4** Dimensions of cross section: $b = 220$ mm, $t = 12$ mm, $h = 600$ mm, $h_1 = 570$ mm, and $V = 200$ kN.

**Solution 5.10-4 Wide-flange beam**




$b = 220$ mm
$t = 12$ mm
$h = 600$ mm
$h_1 = 570$ mm
$V = 200$ kN

MOMENT OF INERTIA (Eq. 5-47)

$$I = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3) = 750.0 \times 10^6 \text{ mm}^4$$

(a) MAXIMUM SHEAR STRESS IN THE WEB (Eq. 5-48a)

$$\tau_{max} = \frac{V}{8It}(bh^2 - bh_1^2 + th_1^2) = 32.28 \text{ MPa} \longleftarrow$$

(b) MINIMUM SHEAR STRESS IN THE WEB (Eq. 5-48b)

$$\tau_{min} = \frac{Vb}{8It}(h^2 - h_1^2) = 21.45 \text{ MPa} \longleftarrow$$

(c) AVERAGE SHEAR STRESS IN THE WEB (Eq. 5-50)

$$\tau_{aver} = \frac{V}{th_1} = 29.24 \text{ MPa} \longleftarrow$$

$$\frac{\tau_{max}}{\tau_{aver}} = 1.104 \longleftarrow$$

(d) SHEAR FORCE IN THE WEB (Eq. 5-49)

$$V_{web} = \frac{th_1}{3}(2\tau_{max} + \tau_{min}) = 196.1 \text{ kN} \longleftarrow$$

$$\frac{V_{web}}{V} = 0.981 \longleftarrow$$

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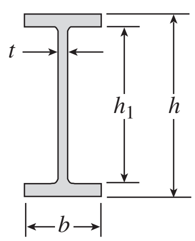
**Problem 5.10-5** Wide-flange shape, W 18 × 71 (see Table E-1, Appendix E); $V = 21$ k.

**Solution 5.10-5 Wide-flange beam**




W 18 × 71
$b = 7.635$ in.
$t = 0.495$ in.
$h = 18.47$ in.
$h_1 = 16.85$ in.
$V = 21$ k

MOMENT OF INERTIA (Eq. 5-47)

$$I = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3) = 1162 \text{ in.}^4$$

(a) MAXIMUM SHEAR STRESS IN THE WEB (Eq. 5-48a)

$$\tau_{max} = \frac{V}{8It}(bh^2 - bh_1^2 + th_1^2) = 2634 \text{ psi} \longleftarrow$$

(b) MINIMUM SHEAR STRESS IN THE WEB (Eq. 5-48b)

$$\tau_{min} = \frac{Vb}{8It}(h^2 - h_1^2) = 1993 \text{ psi} \longleftarrow$$

(c) AVERAGE SHEAR STRESS IN THE WEB (Eq. 5-50)

$$\tau_{aver} = \frac{V}{th_1} = 2518 \text{ psi} \longleftarrow$$

$$\frac{\tau_{max}}{\tau_{aver}} = 1.046 \longleftarrow$$

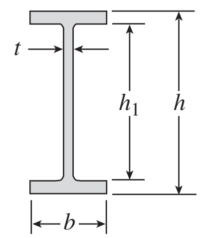
(d) SHEAR FORCE IN THE WEB (Eq. 5-49)

$$V_{web} = \frac{th_1}{3}(2\tau_{max} + \tau_{min}) = 20.19 \text{ k} \longleftarrow$$

$$\frac{V_{web}}{V} = 0.961 \longleftarrow$$

**Problem 5.10-6** Dimensions of cross section: $b = 120$ mm, $t = 7$ mm, $h = 350$ mm, $h_1 = 330$ mm, and $V = 60$ kN.

**Solution 5.10-6** **Wide-flange beam**




$b = 120$ mm
$t = 7$ mm
$h = 350$ mm
$h_1 = 330$ mm
$V = 60$ kN

MOMENT OF INERTIA (Eq. 5-47)

$$I = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3) = 90.34 \times 10^6 \text{ mm}^4$$

(a) MAXIMUM SHEAR STRESS IN THE WEB (Eq. 5-48a)

$$\tau_{max} = \frac{V}{8It}(bh^2 - bh_1^2 + th_1^2) = 28.40 \text{ MPa} \longleftarrow$$

(b) MINIMUM SHEAR STRESS IN THE WEB (Eq. 5-48b)

$$\tau_{min} = \frac{Vb}{8It}(h^2 - h_1^2) = 19.35 \text{ MPa} \longleftarrow$$

(c) AVERAGE SHEAR STRESS IN THE WEB (Eq. 5-50)

$$\tau_{aver} = \frac{V}{th_1} = 25.97 \text{ MPa} \longleftarrow$$

$$\frac{\tau_{max}}{\tau_{aver}} = 1.093 \longleftarrow$$

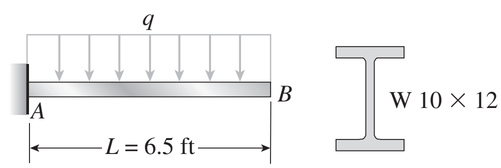
(d) SHEAR FORCE IN THE WEB (Eq. 5-49)

$$V_{web} = \frac{th_1}{3}(2\tau_{max} + 2\tau_{min}) = 58.63 \text{ kN} \longleftarrow$$

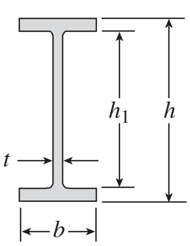
$$\frac{V_{web}}{V} = 0.977 \longleftarrow$$

**Problem 5.10-7** A cantilever beam $AB$ of length $L = 6.5$ ft supports a uniform load of intensity $q$ that includes the weight of the beam (see figure). The beam is a steel W 10 × 12 wide-flange shape (see Table E-1, Appendix E).

Calculate the maximum permissible load $q$ based upon (a) an allowable bending stress $\sigma_{allow} = 16$ ksi, and (b) an allowable shear stress $\tau_{allow} = 8.5$ ksi. (*Note:* Obtain the moment of inertia and section modulus of the beam from Table E-1.)




**Solution 5.10-7** **Cantilever beam**




W 10 × 12
From Table E-1:
$b = 3.960$ in.
$t = 0.190$ in.
$h = 9.87$ in.
$h_1 = 9.87 \text{ in.} - 2(0.210 \text{ in.}) = 9.45$ in.
$I = 53.8 \text{ in.}^4$
$S = 10.9 \text{ in.}^3$
$L = 6.5 \text{ ft} = 78$ in.
$\sigma_{allow} = 16{,}000$ psi
$\tau_{allow} = 8{,}500$ psi

(a) MAXIMUM LOAD BASED UPON BENDING STRESS

$$M_{max} = \frac{qL^2}{2} \quad \sigma = \frac{M_{max}}{S} \quad q = \frac{2S\sigma}{L^2}$$

$$q_{max} = \frac{2S\sigma_{allow}}{L^2} = \frac{2(10.9 \text{ in.}^3)(16{,}000 \text{ psi})}{(78 \text{ in.})^2}$$

$$= 57.33 \text{ lb/in.} = 688 \text{ lb/ft} \longleftarrow$$

(b) MAXIMUM LOAD BASED UPON SHEAR STRESS

$$V_{max} = qL \quad \tau_{max} = \frac{V_{max}}{8It}(bh^2 - bh_1^2 + th_1^2) \quad \text{(Eq. 5-48a)}$$

$$q_{max} = \frac{V_{max}}{L} = \frac{8It(\tau_{allow})}{L(bh^2 - bh_1^2 + th_1^2)}$$

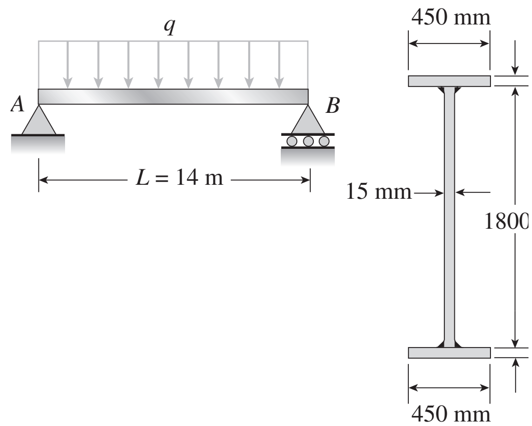
Substitute numerical values:

$$q_{max} = 181.49 \text{ lb/in.} = 2180 \text{ lb/ft} \longleftarrow$$

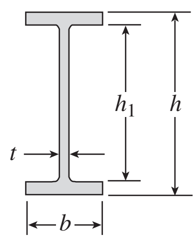
NOTE: Bending stress governs. $q_{allow} = 688$ lb/ft

**Problem 5.10-8** A bridge girder *AB* on a simple span of length $L = 14$ m supports a uniform load of intensity $q$ that includes the weight of the girder (see figure). The girder is constructed of three plates welded to form the cross section shown.

Determine the maximum permissible load $q$ based upon (a) an allowable bending stress $\sigma_{allow} = 110$ MPa, and (b) an allowable shear stress $\tau_{allow} = 50$ MPa.




**Solution 5.10-8** **Bridge girder (simple beam)**




$L = 14$ m
$b = 450$ mm $\quad t = 15$ mm
$h = 1860$ mm $\quad h_1 = 1800$ mm
$\sigma_{allow} = 110$ MPa
$\tau_{allow} = 50$ MPa
$c = h/2 = 930$ mm

Eq. (5-47): $$I = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3)$$

$$= 29.897 \times 10^9 \text{ mm}^4 \longleftarrow$$

$$S = \frac{I}{c} = \frac{29.897 \times 10^9 \text{ mm}^4}{\text{mm}} = 32.147 \times 10^6 \text{ mm}^3$$

(a) MAXIMUM LOAD BASED UPON BENDING STRESS

$$M_{max} = \frac{qL^2}{8} \qquad \sigma = \frac{M_{max}}{S} \qquad q = \frac{8S\sigma}{L^2}$$

$$q_{max} = \frac{8S\sigma_{allow}}{L^2} = \frac{8(32.147 \times 10^6 \text{ mm}^3)(110 \text{ MPa})}{(14 \text{ m})^2}$$

$$= 144.3 \times 10^3 \text{ N/m} = 144 \text{ kN/m} \longleftarrow$$

(b) MAXIMUM LOAD BASED UPON SHEAR STRESS

$$V_{max} = \frac{qL}{2} \qquad \tau_{max} = \frac{V_{max}}{8It}(bh^2 - bh_1^2 + th_1^2) \qquad \text{(Eq. 5-48a)}$$

$$q_{max} = \frac{2V_{max}}{L} = \frac{16It(\tau_{allow})}{L(bh^2 - bh_1^2 + th_1^2)}$$

Substitute numerical values:
$q_{max} = 173.8 \times 10^3$ N/m $= 174$ kN/m $\longleftarrow$

NOTE: Bending stress governs. $\quad q_{allow} = 144$ kN/m

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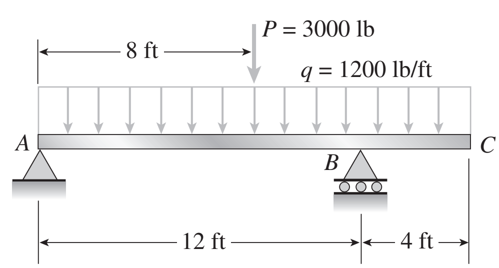
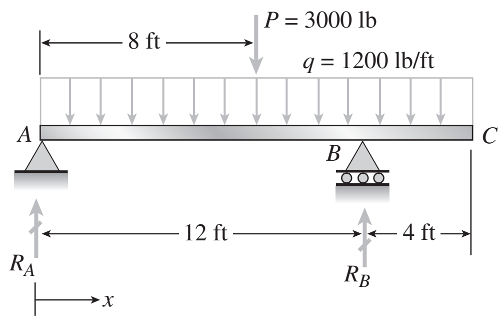
**Problem 5.10-9** A simple beam with an overhang supports a uniform load of intensity $q = 1200$ lb/ft and a concentrated load $P = 3000$ lb (see figure). The uniform load includes an allowance for the weight of the beam. The allowable stresses in bending and shear are 18 ksi and 11 ksi, respectively.

Select from Table E-2, Appendix E, the lightest I-beam (S shape) that will support the given loads.

*Hint:* Select a beam based upon the bending stress and then calculate the maximum shear stress. If the beam is overstressed in shear, select a heavier beam and repeat.



**Solution 5.10-9 Beam with an overhang**




$\sigma_{allow} = 18$ ksi
$\tau_{allow} = 11$ ksi

Select a beam of S shape
$R_A = 7400$ lb
$R_B = 14{,}800$ lb
Maximum shear force: $V_{max} = 10{,}000$ lb at $x = 12$ ft

Maximum bending moment:
$M_{max} = 22{,}820$ lb-ft at $x = 6.167$ ft

REQUIRED SECTION MODULUS

$$S = \frac{M_{max}}{\sigma_{allow}} = \frac{(22{,}820 \text{ lb-ft})(12 \text{ in./ft})}{18{,}000 \text{ psi}} = 15.2 \text{ in.}^3$$

From Table E-2:
Lightest beam is S 8 × 23
$I = 64.9 \text{ in.}^4 \quad S = 16.2 \text{ in.}^3$
$b = 4.171 \text{ in.} \quad t = 0.441 \text{ in.}$
$h = 8.00 \text{ in.} \quad h_1 = 8.00 - 2(0.426) = 7.148 \text{ in.}$

MAXIMUM SHEAR STRESS (Eq. 5-48a)

$$\tau_{max} = \frac{V_{max}}{8It}(bh^2 - bh_1^2 + th_1^2)$$
$$= 3340 \text{ psi} < 11{,}000 \text{ psi} \quad \therefore \text{ ok for shear}$$

Select S 8 × 23 beam ←

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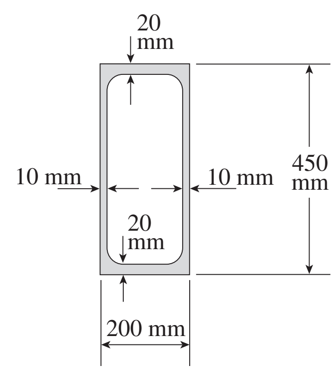
**Problem 5.10-10** A hollow steel box beam has the rectangular cross section shown in the figure. Determine the maximum allowable shear force $V$ that may act on the beam if the allowable shear stress is 36 MPa.

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**Solution 5.10-10 Rectangular box beam**

$\tau_{allow} = 36$ MPa
Find $V_{allow}$

$$\tau = \frac{VQ}{It}$$

$$V_{allow} = \frac{\tau_{allow}It}{Q}$$

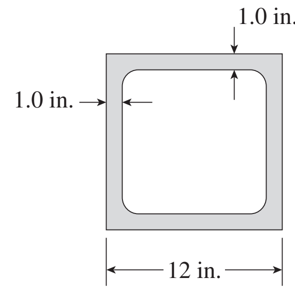
$$I = \frac{1}{12}(200)(450)^3 - \frac{1}{12}(180)(410)^3 = 484.9 \times 10^6 \text{ mm}^4$$

$t = 2(10 \text{ mm}) = 20 \text{ mm}$

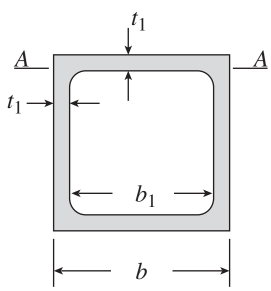
$$Q = (200)\left(\frac{450}{2}\right)\left(\frac{450}{4}\right) - (180)\left(\frac{410}{2}\right)\left(\frac{410}{4}\right)$$
$$= 1.280 \times 10^6 \text{ mm}^3$$

$$V_{allow} = \frac{\tau_{allow}It}{Q}$$
$$= \frac{(36 \text{ MPa})(484.9 \times 10^6 \text{ mm}^4)(20 \text{ mm})}{1.280 \times 10^6 \text{ mm}^3}$$
$$= 273 \text{ kN} \quad \leftarrow$$

**Problem 5.10-11** A hollow aluminum box beam has the square cross section shown in the figure. Calculate the maximum and minimum shear stresses $\tau_{max}$ and $\tau_{min}$ in the webs of the beam due to a shear force $V = 28$ k.




**Solution 5.10-11** **Square box beam**




$V = 28\ k = 28{,}000$ lb

$t_1 = 1.0$ in.

$b = 12$ in.

$b_1 = 10$ in.

$$\tau = \frac{VQ}{It} \qquad t = 2t_1 = 2.0 \text{ in.}$$

Moment of inertia

$$I = \frac{1}{12}(b^4 - b_1^4) = 894.67 \text{ in.}^4$$

Maximum shear stress in the web (at neutral axis)

$$Q = A_1\bar{y}_1 - A_2\bar{y}_2 \qquad A_1 = b\left(\frac{b}{2}\right) = \frac{b^2}{2}$$

$$A_2 = b_1\left(\frac{b_1}{2}\right) = \frac{b_1^2}{2}$$

$$\bar{y}_1 = \frac{1}{2}\left(\frac{b}{2}\right) = \frac{b}{4} \qquad \bar{y}_2 = \frac{1}{2}\left(\frac{b_1}{2}\right) = \frac{b_1}{4}$$

$$Q = \left(\frac{b^2}{2}\right)\left(\frac{b}{4}\right) - \left(\frac{b_1^2}{2}\right)\left(\frac{b_1}{4}\right) = \frac{1}{8}(b^3 - b_1^3) = 91.0 \text{ in.}^3$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(28{,}000 \text{ lb})(91.0 \text{ in.}^3)}{(894.67 \text{ in.}^4)(2.0 \text{ in.})} = 1424 \text{ psi}$$

$$= 1.42 \text{ ksi} \quad \longleftarrow$$

Minimum shear stress in the web (at level $A$-$A$)

$$Q = A\bar{y} = (bt_1)\left(\frac{b}{2} - \frac{t_1}{2}\right) = \frac{bt_1}{2}(b - t_1)$$

$$t_1 = \frac{b - b_1}{2} \qquad Q = \frac{b}{8}(b^2 - b_1^2)$$

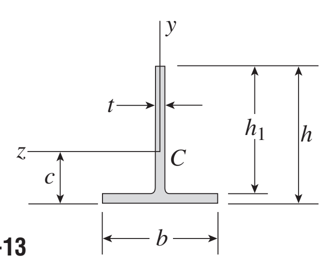
$$Q = \frac{(12 \text{ in.})}{8}[(12 \text{ in.})^2 - (10 \text{ in.})^2] = 66.0 \text{ in.}^3$$

$$\tau_{min} = \frac{VQ}{It} = \frac{(28{,}000 \text{ lb})(66.0 \text{ in.}^3)}{(894.67 \text{ in.}^4)(2.0 \text{ in.})} = 1033 \text{ psi}$$

$$= 1.03 \text{ ksi} \quad \longleftarrow$$

**Problem 5.10-12** The T-beam shown in the figure has cross-sectional dimensions as follows: $b = 220$ mm, $t = 15$ mm, $h = 300$ mm, and $h_1 = 275$ mm. The beam is subjected to a shear force $V = 60$ kN.

Determine the maximum shear stress $\tau_{max}$ in the web of the beam.




**Probs. 5.10-12 and 5.10-13**

**Solution 5.10-12** **T-beam**

$b = 220$ mm $\quad t = 15$ mm $\quad h = 300$ mm

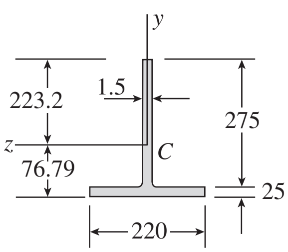
$h_1 = 275$ mm $\quad V = 60$ kN

Find $\tau_{max}$

LOCATE NEUTRAL AXIS
(ALL DIMENSIONS IN MILLIMETERS)

$$c = \frac{\sum A\bar{y}}{\sum A} = \frac{b(h - h_1)\left(\frac{h - h_1}{2}\right) + th_1\left(h - \frac{h_1}{2}\right)}{b(h - h_1) + th_1}$$

$$= \frac{(220)(25)\left(\frac{25}{2}\right) + (15)(275)\left(300 - \frac{275}{2}\right)}{(220)(25) + (15)(275)} = 76.79 \text{ mm}$$




LOCATION OF NEUTRAL AXIS

MOMENT OF INERTIA ABOUT THE $z$-AXIS

$$I_{web} = \frac{1}{3}(15)(223.2)^3 + \frac{1}{3}(15)(76.79 - 25)^3$$
$$= 56.29 \times 10^6 \text{ mm}^4$$

$$I_{flange} = \frac{1}{12}(220)(25)^3 + (220)(25)\left(76.79 - \frac{25}{2}\right)^2$$
$$= 23.02 \times 10^6 \text{ mm}^4$$

$$I = I_{web} + I_{flange} = 79.31 \times 10^6 \text{ mm}^4$$

FIRST MOMENT OF AREA ABOVE THE $z$ AXIS

$$Q = (15)(223.2)\left(\frac{223.2}{2}\right)$$
$$= 373.6 \times 10^3 \text{ mm}^3$$

MAXIMUM SHEAR STRESS

$$\tau_{max} = \frac{VQ}{It} = \frac{(60 \text{ kN})(373.6 \times 10^3 \text{ mm}^3)}{(79.31 \times 10^6 \text{ mm}^4)(15 \text{ mm})}$$
$$= 18.8 \text{ MPa} \quad \longleftarrow$$

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**Problem 5.10-13** Calculate the maximum shear stress $\tau_{max}$ in the web of the T-beam shown in the figure if $b = 10$ in., $t = 0.6$ in., $h = 8$ in., $h_1 = 7$ in., and the shear force $V = 5000$ lb.

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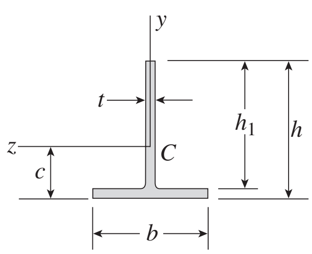
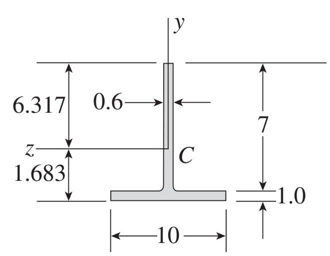
**Solution 5.10-13** **T-beam**




$b = 10$ in.
$t = 0.6$ in.
$h = 8$ in.
$h_1 = 7$ in.
$V = 5000$ lb

Find $\tau_{max}$




LOCATE NEUTRAL AXIS
(ALL DIMENSIONS IN INCHES)

$$c = \frac{\sum A\bar{y}}{\sum A} = \frac{b(h - h_1)\left(\frac{h - h_1}{2}\right) + th_1\left(h - \frac{h_1}{2}\right)}{b(h - h_1) + th_1}$$

$$= \frac{(10)(1)(0.5) + (0.6)(7)(4.5)}{10(1) + (0.6)(7)} = 1.683 \text{ in.}$$

LOCATION OF NEUTRAL AXIS

MOMENT OF INERTIA ABOUT THE $z$-AXIS

$$I_{web} = \frac{1}{3}(0.6)(6.317)^3 + \frac{1}{3}(0.6)(1.683 - 1.0)^3$$
$$= 50.48 \text{ in.}^4$$

$$I_{flange} = \frac{1}{12}(10)(1.0)^3 + (10)(1.0)(1.683 - 0.5)^2$$
$$= 14.83 \text{ in.}^4$$

$$I = I_{web} + I_{flange} = 65.31 \text{ in}^4.$$

FIRST MOMENT OF AREA ABOVE THE $z$ AXIS

$$Q = (0.6)(6.317)\left(\frac{6.317}{2}\right) = 11.97 \text{ in.}^3$$

MAXIMUM SHEAR STRESS

$$\tau_{max} = \frac{VQ}{It} = \frac{(5000 \text{ lb})(11.97 \text{ in.}^3)}{(65.31 \text{ in.}^4)(0.6 \text{ in.})} = 1530 \text{ psi} \quad \longleftarrow$$